Homework for General Physics II set 2 Answer Keys

1. **(Hecht’s 10.8)A narrow single slit (in air) in an opaque screen is illuminated by infrared from a He-Ne laser at 1152.2 nm, and it is found that the center of the tenth dark band in the Fraunhofer pattern lies at an angle of off the central axis. Please determine** **the width of the slit.**

Answer:

According to the irradiance resulting from an idealized coherent line source in the Fraunhofer approximation:

( hecht used beta, I used alpha, just the difference in symbols)

Where

The center of the tenth dark band in the Fraunhofer pattern:

The width of the slit:

**At what angle will the tenth minimum appear if the entire arrangement is immersed in water (****) rather than air ()?**

Answer:

2. (Modified Hecht’s 10.14) Show that If the incoming light is a plane wave normal to the diffraction screen, Fraunhoffer diffraction pattern (light intensity) on the observing screen has a center of symmetry, ; x’ and y’ are coordinates of the observing screen; regardless of the configuration of the aperture as long as there are no phase variations in the field over the region of the hole. This restriction is equivalent to saying the aperture function is real. ( In short, this problem asks you to prove for any real aperture functions, the F-diffraction pattern has center of symmetry)

Answer: x-y is diffraction screen and x’-y’ is observing screen, they are separated by a distance Z0 or under paraxial condition Z0=r0, r0 is the distance from center of diffraction screen (origin of x-y plane) to a point (x’,y’) on observing screen. Under F-diffraction the light **field** at (x’, y’) is:



Where  is the incoming field distribution on the diffraction screen, and in normal illumination it is just a constant amplitude u0, t(x,y) is the aperture function of the diffraction screen which may affect the incoming field distribution. (For the examples discussed in the class, we have simple t(x,y), t(x,y)=1 at the opening, 0 elsewhere; such aperture are called black-white type) Under the condition of the problem, we know t(x,y) is a real function meaning the aperture may alter the amplitude but not the phase of the incoming field.

The complex conjugate is:



The intensity is then:

We see right away it has center of symmetry, if you replace x’, y’ by –x’, -y’, the two integrations are just interchanged and the product is same. (the integration is over x-y screen and x’,y’ are just some parameters in the integration)

3. For a single slit (in 1-D) with opening width of 3*a*, you should be able to write out the field distribution on observing screen for Fraunhoffer diffraction; Now consider we insert an **opaque block** with width *a* at the center of the original slit (this will block 1/3 of the original slit), please prove that the F-Diffraction pattern for the blocked slit is in form of ,  .

Of course I will prove this in multi-slits diffraction, but here I’d like you to work it out with the knowledge of the single slit diffraction formula. Hint: Thinking the following questions: what is the field distribution U1 on the observing screen that is just for the slit with 3*a* width? What is the field distribution U2 which is for a slit with width *a*? Then what is the field of blocked slit ? Knowing the field, I am sure you can find the intensity. (Noticed here too superposition of the field not the intensity)

Answer:



C is some constant in the calculation.





Using formula: ,



We can call the  which is the intensity of the maximum at angle 0.

Incidentally this maximum intensity I0 is smaller that the maximum of 3a opening (which is ) which make sense; and is 4 times as big as slit with width a.

We will find a better or at least more systematic method of evaluating such diffraction pattern in multi-slits diffraction.

4. For an opening slit with width *a*, we cover the opened part with an absorbing material which will change the amplitude of the illuminating field by , x=0 is the center of the slit. The incoming light is a plane wave with wavelength  at normal angle. What is Fraunhoffer diffraction pattern on the observing screen? Write out the field distribution first and then intensity on the observing screen. Please also make a sketch of the field distribution for this slit and compare it with the single open slit.

Answer:

We need to evaluate the integral:



I will call the front combination is some constant C; also 

Then after cosmetic:



At least two ways to do this integration either by change cosine into Euler complex exponential or change complex exponential into cosine and sine, I will prefer changing cosine into Euler exponential:



We can further make cosmetic to let it look more like sinc function by times *a* and divided *a:*



So in terms of  (we shall see this corresponds to spatial frequency of the diffraction screen in Fourier transform later), the two parts in the field are just single slit field distribution but 1) with amplitude halved 2) shifted by +K0 and –K0

Or in terms of , also remember :



The sketch is omitted here, it is just two sinc functions centered not at 0, but at + and -, with amplitude halved compare with single open slit case.

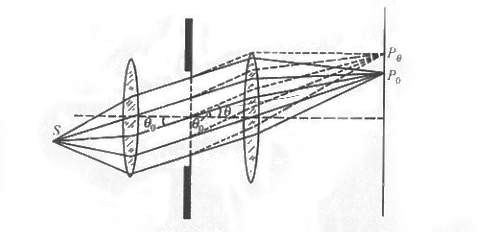
The intensity is just square of the U and its form is straightforward.

5. Zhao’s book, problem 1, pg 224. (the single slit Fraunhoffer diffraction illuminated by a inclined plane wave, i.e the incoming light is a plane wave, but its direction has an angle θ with respect to the normal of the single slit diffraction screen, find the expression for the Fraunhoffer diffraction pattern )



I just copy the answer:

1. **（《光学》赵凯华 上册第二章习题1 pg.224）平行光以角斜入射在宽度为a的单缝上，求证：**

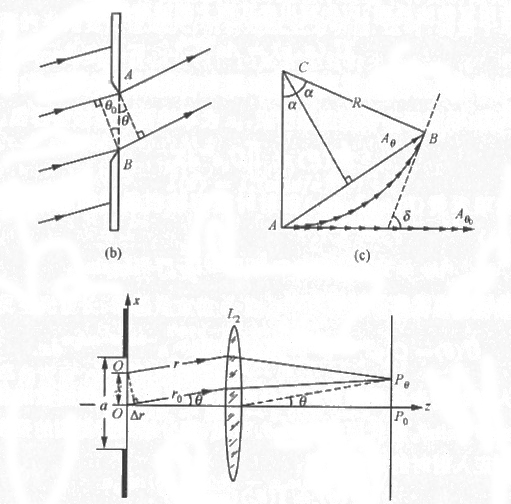
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1. **夫琅禾费衍射的强度公式基本不变（忽略倾斜因子），即**

**式中为零级中心强度，只不过的定义与正入射不同：**

解答：

与正入射情况相比，斜入射时光孔面上各点次波源的相位是不同的。因此，在分析到达场点的各次级扰动之间的相位关系时，不仅要考虑后场光程差的影响，而且还要考虑前场光程差的影响。



若用矢量图解法处理，代表边缘两点A, B到达场点的相位差应为：

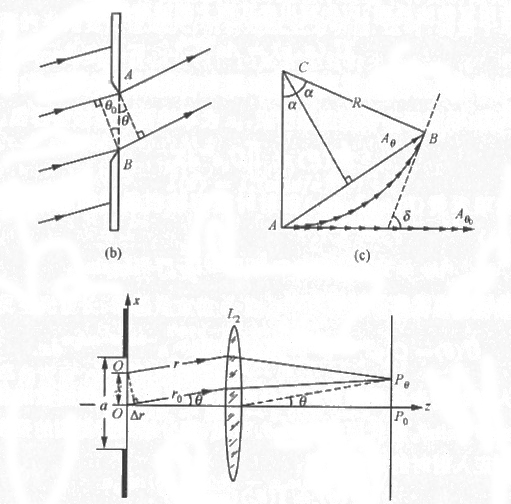
其它运算与正入射相同，于是强度分布函数成为：

式中

处理，应先写出光孔面上的波前函数 (I prefer this way)

考虑到傍轴条件下的菲涅耳-基尔霍夫衍射积分公式为：

并在夫琅禾费衍射时改写被积函数的相因子。



于是单缝夫琅禾费衍射场为：

与正入射时候的夫琅禾费衍射积分公式相比，仅仅在被积函数的相因子中由替代原来的。可见，积分结果所得到的强度分布函数与正入射时候的形式相同：

只是

1. **零级中心的位置在几何光学像点处；**

解答：

根据强度分布公式，零级（极强）出现在的地方。此时，各衍射线（连同入射光线）之间无光程差，根据费马原理推得的物像等光程性，这正是几何光学像点的位置。

1. **零级斑半角宽度为：**

解答：

令，得到零级两侧第一暗点的衍射角满足：

设零级半角宽度为，则：

注意到很小，用微分运算近似，得：

即：

由此可见，在缝宽a不变的条件下，照明单缝的平行光的倾角越大，零级斑的半角宽度也越大，衍射效应更为明显。这时，相当于单缝有效宽度变小了。

1. **如果考虑到单缝两侧并非同一介质，情况将怎样？**

解答：

如果衍射屏前后两侧为不同介质，设前场照明空间折射率为n1，后场衍射空间折射率为n2，则衍射强度分布函数形式不变，仍为：

式中

为真空光波长。零级角方位由给出，它也满足折射定律：

零级半角宽度由下式确定：

即：

取微分近似，得：

6. Hecht’s 10.27

 is the spacing of dots which is 1/10 inch (or 0.25cm), and for people stand at a distance d; the angular separation of the dots is:



This angle has to be **smaller** than the resolution of the eye which is  for 4.0 mm pupil and at , in order to have the effect of blending or blur (meaning our eyes cannot tell the dots apart).



This is of course under the ideal eye resolution. In reality resolution for a common human eye is about 1/1000 as mentioned in Hecht’s book, if so then the d will be 100 inches.

**7. (Hecht’s 10.28)The Mount Palomar telescope has an objective mirror with a 508-cm diameter. Determine its angular limit of resolution at a wavelength of 550nm, in radians, degrees, and** **seconds of arc.**

Answer:

Due to Rayleigh criterion:

**How far apart must two objects be on the surface of the Moon if they are to be resolvable by the Palomar telescope? The Earth-Moon distance is; take.**

Answer:

**How far apart must two objects be on the Moon if they are to be distinguished by the eye? Assume a pupil diameter of 4.00 mm.**